

Gravitational waves receiver

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Introduction

Attempts to register space gravitational waves have been made in a number of countries for more than a quarter of a century. Different modifications of Weber's massive mechanical oscillators have been used for this purpose [1 - 3]. The importance of these studies consist in the attempt to confirm, first of all, one of the most essential corollaries of the General relativity theory and, secondly, to provide new channels for gaining information on the Universe. It is thus necessary to design a highly sensitive receiver for the registration of weak gravitational waves which are transmitted by a hypothetical space sources and reach the Earth. The radio-physical foundations of this problem are laid down by the school of V.B. Braginsky [4,5].

The present paper tackles the problem of further improvement of gravitational wave receivers by increasing their band range along with keeping of increasing their sensitivity. A resonance gravitational waves receiver is considered. A mechanical oscillator with a quality factor $Q_s \approx 10^7 - 10^{10}$ is used as an aerial sensor, which provides a high level of the eigen fluctuation noises. The expected frequency range of the basis types of space gravitational sources is $f \leq 10^3 - 10^4$ Hz. The information on the shape of the expected signal is of special interest for Astrophysics. Hence, parallel to the receiver sensitivity, the problem of realizing a wide band range recording is of special importance.

Diagram of the gravitational waves receiver

The diagram of the gravitational wave receiver is shown in Fig. 1, consisting of: HQMO (GS) - high quality mechanical oscillator (gravitational sensor); 4-FEMPS - 4-frequency electro-mechanical parametric system;

4-FPA - 4-frequency parametric amplifier; PG - high frequency pumping generator; DPA - degenerate parametric amplifier; FC (x2) - frequency converter (doubler); SRO - system for oscillations registration.

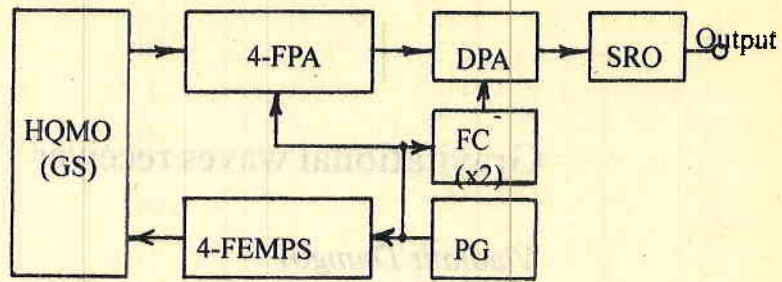


Fig.1. Diagram of the gravitational wave receiver

4-FEMPS is to compensate the elasticity of the gravitational sensor HQMO (GS), which provides practically an frequency independence of the impedance and of the transmission coefficient in the input circuit of the gravitational wave receiver within its working frequency band range. 4-FPA provides a low noise amplification of the received signal in a regime, optimal with respect to noise. DPA is a second parametric amplifier which, together with performing a low-noise amplification, damps down the noise from the lateral frequency bands of 4-FPA. PG provides a direct high frequency supply of 4-FEMPS and 4-FPA. It supplies DPA, too, through FC (x2). The frequency of the PG voltage is much higher than the frequency range of the expected gravitational waves. The concluding unit SRO of the gravitational wave receiver performs an adaptive damping of the HQMO (GS) eigen free oscillations, and the signal sought is to be identified on such a background. Parallel to the basic problem, SRO is a system for damping down seismic, electromagnetic and other noise effects, disturbing the receiver functioning.

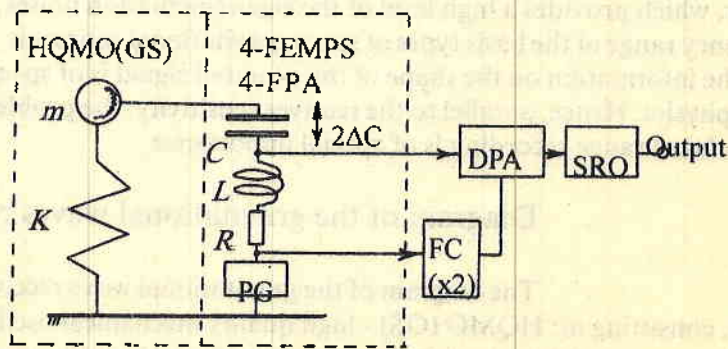


Fig.2. An idea of the design of a gravitational wave receiver

To state things more clear, the idea of the design of a gravitational wave receiver is given in Fig.2. A mechanical oscillator with mass m and differential coefficient of elasticity K stands for HQMO (GS). The transformation of the mechanical oscillations into electrical ones is performed by a capacitive sensor C , directly switched to HQMO (GS). The capacitive sensor C is an element of the oscillating system CLR which, together with the pumping generator PG and on line with HQMO (GS) realizes negative elasticity and low-noise parametric amplification, combining the functions of 4-FEMPS and 4-FPA. For a definite set-up of 4-FEMPS, its reaction to the mechanical modulation of C is expressed by an introduction of an equivalent negative elasticity and an equivalent friction in the mechanical oscillator HQMO (GS) [4,6-9]. In this case 4-FPA lacks a definite parametric element, whose parameter should vary with the high pumping frequency, as is assumed in the theory of parametric systems. However, the modulating effect of HQMO (GS) through the capacitive sensor C causes the generation of summary and difference combination frequencies, which determine the CLR system character as a 4-frequency parametric amplifier (4-FPA). DPA can be designed by employing a bridge-balance circuit with high positive input impedance and low eigen fluctuation noises [9-11]. One of the SOR possible configurations is given in [3].

Analysis of the noise characteristics

The well known electro-mechanical analogies [12] can be used to perform the analysis. They allow for the reduction of the problem to a pure 'electrical' problem [3]. Such analogies are: the electro-motive force \leftrightarrow mechanical force, inductance \leftrightarrow mass, capacitance \leftrightarrow elasticity, resistance \leftrightarrow friction, etc.

An equivalent circuit of the gravitational wave receiver, together with the noise sources, is given in Fig. 3. The equivalent gravitational impact (IA) on the oscillator mass is expressed by the equivalent conductance of the irradiation resistance G_G . The gravitational sensor HQMO (GS) is presented by the reactive parameters L_s, C_s , and by the conductance of friction losses G_s . 4-FEMPS is represented by the equivalent conductance G_g , and by the equivalent negative capacitance C_g introduced in the input circuit of the gravitational wave receiver. The presence of 4-FPA is outlined by the general complex admittance Y_p and by the complex admittance Y_{add} , added due to the reversibility of the modulation - parametric interactions in 4-FPA[11, 13]. Since a large coefficient of amplification of 4-FPA, as well as of DPA, is expected to be attained, DPA is presented, together with SOR, as an amplifying two-port unit (load) ATP (L), with an input general complex admittance Y_L .

The noise-generating properties of IA, HQMO (GS), 4-FPA and ATP

(L) are characterized by equivalent effective values of noise electric current generators $\sqrt{i_{NGS}^2}$, $\sqrt{i_{NP}^2}$, $\sqrt{i_{NL}^2}$ by a noise voltage $\sqrt{U_{NL}^2}$ and by a noise correlation admittance \dot{Y}_{cor} , which are determined by the following expressions [14,15] :

$\overline{i_{NGS}^2} = G_{GS} N$, $G_{GS} = G_G + G_S$, $\overline{i_{NP}^2} = \text{Re} \dot{Y}_p N$, $\overline{i_{NL}^2} = G_{NL} N$, $\overline{U_{NL}^2} = R_{NL} N$,
 $\dot{Y}_{cor} = \left(\sqrt{\overline{i_{NL}^2}} \sqrt{\overline{U_{NL}^{2*}}} \right) / \left| \overline{U_{NL}^2} \right|$, where $N = 4\kappa T \Delta f$, κ is the Boltzmann constant, T is the absolute temperature (for HQMO (GS), $T \equiv T_S$ for 4-FPA - $T \equiv T_p$, for ATP (L) - $T \equiv T_L$, Δf is the frequency band, G_{NL} and R_{NL} are, respectively, the equivalent noise conductance and the equivalent noise resistance of the amplifying two-port unit (load) ATP (L)). The mark "*" denotes a complex-conjugate quantity. The possibility of introducing additional noise in HQMO (GS) under the performance of 4-FEMPS is taken into account by including the noise generator of electric current $\sqrt{i_{NG}^2}$. The additional noise, due to the reversibility of the interactions in 4-FPA (i.e. due to the mutual combinational noise transformation in the 4 working frequency bands [11,13]) is expressed by means of the current source $\sqrt{i_{Nadd}^2}$. The flicker noise (noise of type "1/f") can be neglected and their relative weight is considered to be negligible, since, in 4-frequency parametric systems, they are present as a rule in the infra-low frequency region only, till units Hz [3,9].

It is not difficult to show that the use of 4-FEMPS expands significantly the receiver frequency band as a result of the capacitance C_s full compensation (i.e. compensation of the elasticity of the gravitational sensor). For a positive detuning $\xi = \frac{\omega_{p_0}}{2Q_p}$ of the oscillating system CLR, where ω_{p_0} is the system resonance frequency and Q_p is its quality factor, an equivalent differential elasticity is introduced in HQMO (GS) [4], with a coefficient $\Delta K_p = -\frac{U_o^2 Q_p^2 S}{16\pi d_o^3}$. Here U_o is the amplitude of the supplying voltage, produced by the pumping generator PG, S and d_o are the area of the capacitor plates (the capacitive sensor) and the initial distance between them, respectively. It is obvious that increasing the amplitude U_o for other intact conditions, a full compensation of HQMO (GS) eigen differential elasticity can be attained. The problem is now to estimate the effective use of 4-FEMPS, regarding the possibility of keeping the sensitivity of the gravitational wave receiver within the significantly expanded frequency band.

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strongly expressed resonance characteristic. The response of a system with such an input selectivity to an arbitrary signal is always a sinusoid with a frequency, equal to the resonance frequency of HQMO (GS) and radically different from the form of the acting gravitational wave. The processing of the signal at the receiver output by using such a frequency filter at the input is a difficult task. Really, the output signal is practically determined by the free oscillations of HQMO (GS), which are oscillations of frequency ω_{so} and with random amplitude and phase. The difference between the real record and the expected one, corresponding to undisturbed free oscillations, is used in order to isolate the signal, carrying information about the acting gravitational force. During the receive of a wide-band signal, the damping of the non-resonance part of its spectrum is of order Q_s (Q_s is the HQMO (GS) quality factor). However, too high and practically unattainable accuracy of the record is needed in order to isolate the whole spectrum. For instance, a record with 10^7 amplitude gradations is needed to receive the whole signal spectrum, for $Q_s = 10^6$. This corresponds to a 23-digit binary code. Modern gravitational sensors have as a rule $Q_s \sim 10^{10}$. Hence, an equalization of the amplitude-frequency characteristic is to be performed in order to receive and process a wide-band range signal. The canonical approach to the solution of this problem, in accordance with the detection theory, states to use an optimal filter. If the character of the expected signal is that of white noise, the optimal filter is to possess an amplitude-frequency characteristic opposite to that of HQMO (GS). Such is rejecting filter with the same quality factor Q_s and its design is practically difficult, due to the impossibility of satisfying the requirements for parameter stability.

We consider a simpler technical solution. HQMO (GS) frequency characteristics of transmission can be given in the following form:

$$k_{GS}(\xi) = \frac{\xi Q_s}{\sqrt{1 + Q_s^2(1 - \xi)^2}},$$

where $\xi = \frac{\omega_s}{\omega_{so}}$, ω_s is the signal frequency of the range of gravitational waves expected. The frequency transfer characteristic, out of the HQMO (GS) band of transmission (for frequencies $\xi \geq 1 \pm \frac{1}{Q_s}$ for example), can be approximately expressed as $k_{GS}(\xi) \cong \frac{\xi}{|1 - \xi|}$ (e.g., the formular error for $\xi = 1 \pm \frac{1}{Q_s}$ is 0,5%). Thus, the problem of equalizing the frequency characteristic can be solved by using two independent operations: (i) a removal of the peak in the transition band $\frac{\omega_{so}}{Q_s}$ of HQMO (GS), and (ii) a correction of the frequency characteristic. The first operation supposes damping of the HQMO (GS) eigen free oscillations

and this is the most often performed in SOR by the use of a self-adapting system which compensates oscillations with frequency ω_{so} in the output units of the gravitational wave receiver. The compensation is realized on the basis of a generated oscillation with the same frequency and amplitude and opposite phase (see [3]). The second operation is supposed to be performed at input of the gravitational wave receiver. Here we mean the compensation of the capacitance of the gravitational wave sensor (see Fig.3), when $\omega(C_s - C_{g-}) = 0$. Then transmission coefficient of HQMO (GS) is close to 1 and it can be specified as follows:

$$k_{GS}(\omega) = \frac{1}{\sqrt{1 + [\omega L_s(G_G + G_S + G_g)]^2}} \cong 1$$

since the following condition is fulfilled: $\omega L_s(G_G + G_S + G_g) \ll 1$, which is related to the extremely weak interaction between the gravitational wave and matter ($\omega L G_G \ll 1$) and to the high quality factor of the gravitational sensors ($\omega L_s G_S \ll 1$).

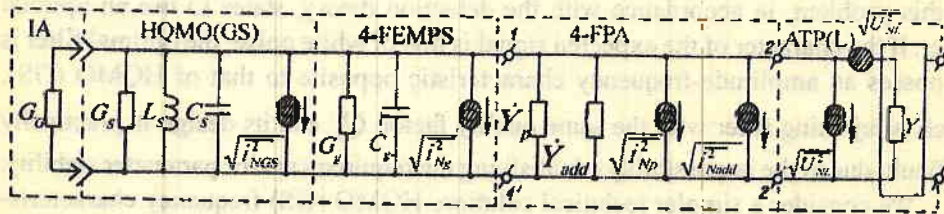


Fig.3. Equivalent impedance - noise circuit of gravitational wave receiver

Since the system lacks impedance matching, the noise coefficient of the circuit in Fig.3 can be determined as

$$(1) \quad F = 1 + \frac{\overline{i_{NE}^2}}{\overline{i_{NGS}^2}},$$

where

$$\overline{i_{NE}^2} = \overline{i_{Ng}^2} + \overline{i_{Np}^2} + \overline{i_{Nadd}^2} + \overline{i_{NL}^2} + \overline{U_{NL}^2} \left| \frac{\dot{Y}_L (\dot{Y}_{GS} + \dot{Y}_g + \dot{Y}_p + \dot{Y}_{add})}{\dot{Y}_L + \dot{Y}_{GS} + \dot{Y}_g + \dot{Y}_p + \dot{Y}_{add}} \right|^2 - 2\overline{U_{NL}^2} \left| \dot{Y}_{cor} (\dot{Y}_{GS}^* + \dot{Y}_g^* + \dot{Y}_p^* + \dot{Y}_{add}^*) \right| \left| \frac{\dot{Y}_L}{\dot{Y}_L + \dot{Y}_{GS} + \dot{Y}_g + \dot{Y}_p + \dot{Y}_{add}} \right|^2,$$

\dot{Y}_s is the total complex admittance of IA and HQMO (GS), \dot{Y}_g is the total complex admittance introduced by 4-FEMPS into HQMO (GS). The expression (1) can be rewritten as

(2)

$$F = 1 + \frac{G_{NL}}{G_{GS}} + F_{4-FEMPS} \frac{G_g}{G_{GS}} + \frac{\overline{U_{NP}^2} |\dot{Y}_{GS} + \dot{Y}_g + \dot{Y}_p + \dot{Y}_{add}|^2}{G_{GS} N} + \frac{R_{NL}}{G_{GS}} \left[|\dot{Y}_{GS} + \dot{Y}_g + \dot{Y}_p + \dot{Y}_{add}|^2 - 2 |\dot{Y}_{cor} (\dot{Y}_{GS}^* + \dot{Y}_g^* + \dot{Y}_p^* + \dot{Y}_{add}^*)| \right],$$

where $F_{4-FEMPS}$ is the noise coefficient of 4-FEMPS, which can be expressed here as $F_{4-FEMPS} = \frac{G_{4-FEMPS}}{G_g} = \frac{i_{Ng}^2}{4\kappa T_s \Delta f G_g}$, $G_{4-FEMPS}$ is the equivalent noise conductance of 4-FEMPS, $\overline{U_{NP}^2} = \frac{i_{Np}^2 + i_{Nadd}^2}{|\dot{Y}_{GS} + \dot{Y}_g + \dot{Y}_p + \dot{Y}_{add}|^2}$ is the noise voltage, introduced by 4-FPA.

Upon a condition that 4-FEMPS realizes a total compensation of the capacitance C_s in HQMO (GS) and considering the negligible effect of the inductance L_s in a wide frequency band $\omega > \omega_{so} \left(1 + \frac{1}{2Q_s} \right)$, the noise coefficient of the considered gravitational wave receiver takes the form

(3)

$$F = 1 + \frac{G_{NL}}{G_{GS}} + F_{4-FEMPS} \frac{G_g}{G_{GS}} + \frac{\overline{U_{NP}^2} (G_{GS} + G_g + G_p + G_{add})^2}{4\kappa T_s \Delta f G_{GS}} + \frac{R_{NL}}{G_{GS}} \left[(G_{GS} + G_g + G_p - G_{add})^2 - 2 |\dot{Y}_{cor} (\dot{Y}_{GS} + G_g + G_p - G_{add})| \right].$$

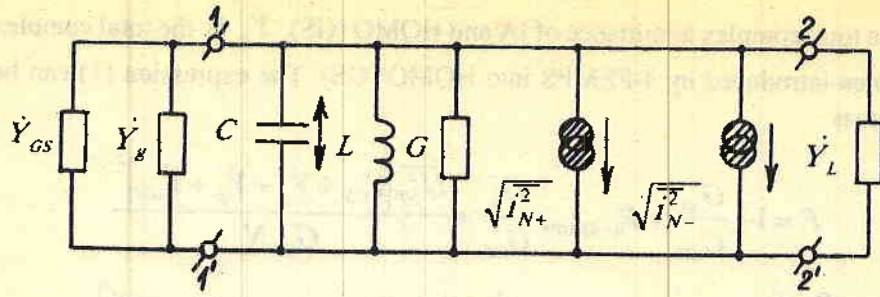


Fig.4. Equivalent impedance - noise circuit of the 4-frequency parametric amplifier (4-FPA)

To determine $\overline{U_{Np}^2}$ we use the impedance-noise equivalent circuit of 4-FPA, given in Fig.4, where G is the referred active conductance to the resistance R (see Fig.2). Considering that the capacitance C (Fig.2) is modulated by the gravitational sensor HQMO (GS) with an amplitude $2\Delta C$ and frequency ω_s of the band of the expected gravitational waves, the electric current flowing through C can be linked with voltage onto it through the following complex matrix equation:

$$(4) \quad \begin{pmatrix} \dot{I}_p \\ \dot{I}_+ \\ \dot{I}_- \end{pmatrix} = \begin{pmatrix} \dot{Y}_{p\Sigma} & j\Delta C\omega_p & j\Delta C\omega_p \\ j\Delta C\omega_+ & \dot{Y}_{\Sigma+} & 0 \\ j\Delta C\omega_- & 0 & \dot{Y}_{\Sigma-} \end{pmatrix} \begin{pmatrix} \dot{U}_p \\ \dot{U}_+ \\ \dot{U}_- \end{pmatrix}$$

Here \dot{I}_p and \dot{U}_p are the complex amplitudes of the current and the voltage at the pumping frequency ω_p , under which PG is functioning, \dot{I}_+ , \dot{U}_+ and \dot{I}_- , \dot{U}_- are the complex amplitudes of the currents and the voltages at the summary ($\omega_+ = \omega_p + \omega_s$) and difference ($\omega_- = \omega_p - \omega_s$) combination frequency, respectively.

The admittance $\dot{Y}_{p\Sigma}$, $\dot{Y}_{\Sigma+}$ and $\dot{Y}_{\Sigma-}$ are in fact the total admittance of the circuit in Fig.4 at the frequencies ω_p , ω_+ and ω_- , respectively.

If we express the currents \dot{I}_p , \dot{I}_+ and \dot{I}_- and the voltage in the matrix equation (4) as noise currents (see Fig.4) $\dot{I}_p \rightarrow \sqrt{\overline{i_{Np}^2}}$, $\dot{I}_+ \rightarrow \sqrt{\overline{i_{N+}^2}}$, $\dot{I}_- \rightarrow \sqrt{\overline{i_{N-}^2}}$ and a noise voltage $U_p \rightarrow \sqrt{\overline{U_{Np}^2}}$, we obtain

$$(5) \quad \overline{U_{Np}^2} = \overline{i_{Np}^2} \left| \frac{D_p}{D} \right|^2 + \overline{i_{N+}^2} \left| \frac{D_+}{D} \right|^2 + \overline{i_{N-}^2} \left| \frac{D_-}{D} \right|^2,$$

where D is the determinant of the matrix in (4), and D_p , D_+ and D_- are the

respective algebraic complements.

Considering that the generalized input admittance of 4-FPA is expressed from the matrix equation (4) as

$$(6) \quad \dot{Y}_{in} = \frac{I_p}{U_p} = \frac{D}{D_p},$$

we obtain the following expression for the total input noise current $\overline{i_{N4-FPA}^2}$ of 4-FPA:

$$(7) \quad \overline{i_{N4-FPA}^2} = \overline{U_{Np}^2} |\dot{Y}_{in}|^2 = \overline{i_{Np}^2} + \overline{i_{N+}^2} \left| \frac{D_+}{D_p} \right|^2 + \overline{i_{N-}^2} \left| \frac{D_-}{D_p} \right|^2,$$

i.e., the additional noise current, introduced by 4-FPA, is

$$(8) \quad \overline{i_{Nadd}^2} = \overline{i_{N+}^2} \left| \frac{D_+}{D_p} \right|^2 + \overline{i_{N-}^2} \left| \frac{D_-}{D_p} \right|^2.$$

It is obvious that the sum of the admittance \dot{Y}_p and \dot{Y}_{add} , given in Fig.3, are determined as: $\dot{Y}_p + \dot{Y}_{add} = \dot{Y}_{in} - (\dot{Y}_{GS} + \dot{Y}_g + \dot{Y}_L)$

Estimations and conclusions

The paper discusses the idea for eliminating the radical defect of the gravitational wave receiver, that is, the receiver extremely narrow band, imposed by the requirement for an extremely high sensitivity. The study proposes to eliminate this defect by performing a compensation of the capacitance (the differential elasticity) of the gravitational sensor through a negative capacitance (a negative differential elasticity). The latter is to be created and introduced in the gravitational sensor by a 4-frequency electro-magnetic parametric system.

An approach to study the receiver noise characteristics is outlined. However, the following considerations are to be additionally laid down.

Parallel to the negative differential elasticity, positive effective friction is introduced in the mechanical oscillator of the gravitational sensor (that is shown in [4,16]): $H_0 = \frac{SU_o^2 Q_p^4}{16\pi d^3 \omega_p}$, which determines the occurrence of an equivalent 'cooling' in the mechanical sensor. The essence of this effect, however, consists

in the fact, that the effective noise temperature of the mechanical oscillator decreases equivalently and becomes: $T_{s\text{eff}} = \frac{T_s}{Q_p}$. In the terms of the 'electrical language' the spectral density of the introduced additional noise current $\overline{i_g^2}$ in the sensor equivalent circuit (Fig.3) is given by: $\overline{i_g^2} \approx \kappa T_s G_g \frac{\omega_s}{\omega_p^2}$. For instance, for $\omega_p = 2\pi 10^{10} \text{ s}^{-1}$ and $\omega_s = 2\pi \text{ s}^{-1}$ the introduced noise current $\overline{i_g^2}$ will be negligible. It can be shown, on the basis of equations (4), (5) and (8), that the detuning of the 4-frequency electromechanical parametric system, creating negative capacitance (negative elasticity), causes a negligible increase of the eigen fluctuation noise. A separate problem is the design of a super stable 4-frequency electro-mechanical parametric system for creating negative capacitance (negative differential elasticity). This can be done, for instance, by using a super conducting resonator and self-adapting system for a frequency autotuning.

The paper outlines only a way for exploring and analyzing such a perspective - the application of reactive compensation in the gravitational, high quality, mechanical sensor, in order to widen the receiving frequency band without spoiling the receiver sensitivity. It is obvious that a special attention is to be paid to the limit noise characteristics, including a full specification of all active factors. Employing the outline approach, an opposite transition from electric to mechanical values is to be performed by using electro-mechanical analogies. Only then can the mechanical gravitational sensor be entirely (and quantitatively) specified.

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Приемник на гравитационни вълни

Владимир Дамгов

(Резюме)

С цел силно разширяване на честотната лента на приемника на гравитационни вълни без съществено влошаване на неговата чувствителност е предложено да се компенсира капацитетът (диференциалната еластичност) на гравитационния датчик с помощта на отрицателен капацитет (отрицателна диференциална еластичност), произведена от 4-честотна електромеханична параметрична система. Набелязан е подход за изследване на шумовите характеристики на приемник на гравитационни вълни, съставен от висококачествен механичен осцилатор, 4-честотна електромеханична параметрична система, 4-честотен параметричен усилвател, изроден параметричен усилвател и система за регистрация на трептенията. Изследванията на отделни елементи (компоненти) и релативни физически ефекти, проведени от различни автори, са обединени в една обща концепция за разработване на широколентов високочувствителен приемник на гравитационни вълни.